Coordinate geometry and making connections

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Choose four points and join them to form a quadrilateral.

Find the midpoint of each line segment and join these to form a quadrilateral.

What type of quadrilateral have you formed?

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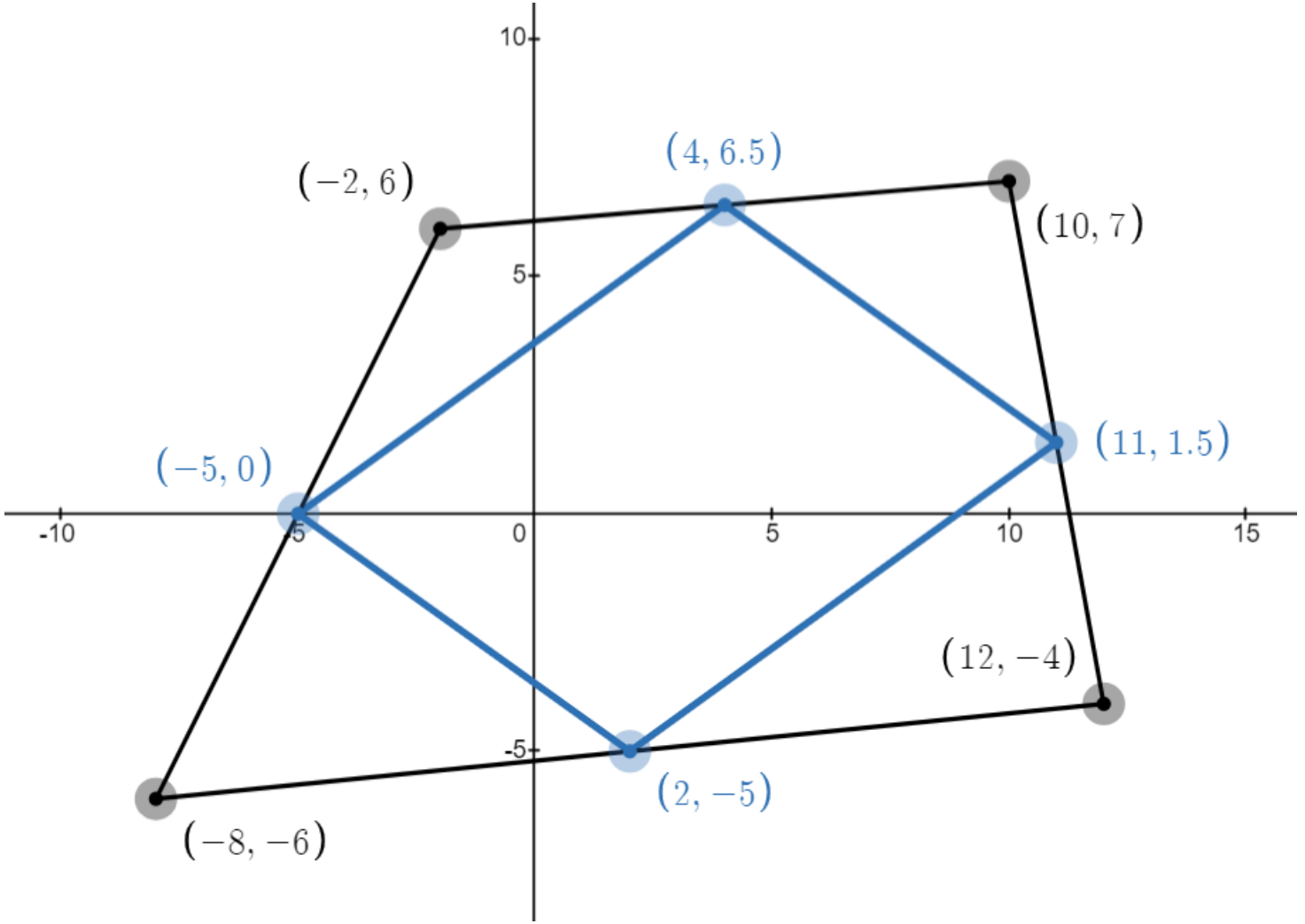
Find the midpoint of each line segment and join these to form a quadrilateral.

What type of quadrilateral have you formed?



$$\begin{pmatrix} 4 \\ 6.5 \end{pmatrix} - \begin{pmatrix} -5 \\ 0 \end{pmatrix} = \begin{pmatrix} 9 \\ 6.5 \end{pmatrix} \qquad \begin{pmatrix} 11 \\ 1.5 \end{pmatrix} - \begin{pmatrix} 2 \\ -5 \end{pmatrix} = \begin{pmatrix} 9 \\ 6.5 \end{pmatrix}$$

General formula for the midpoint:

$$midpoint = \left(\frac{x_1 + x_2}{2}\right), \left(\frac{y_1 + y_2}{2}\right)$$

$$\frac{1 + n}{2}$$

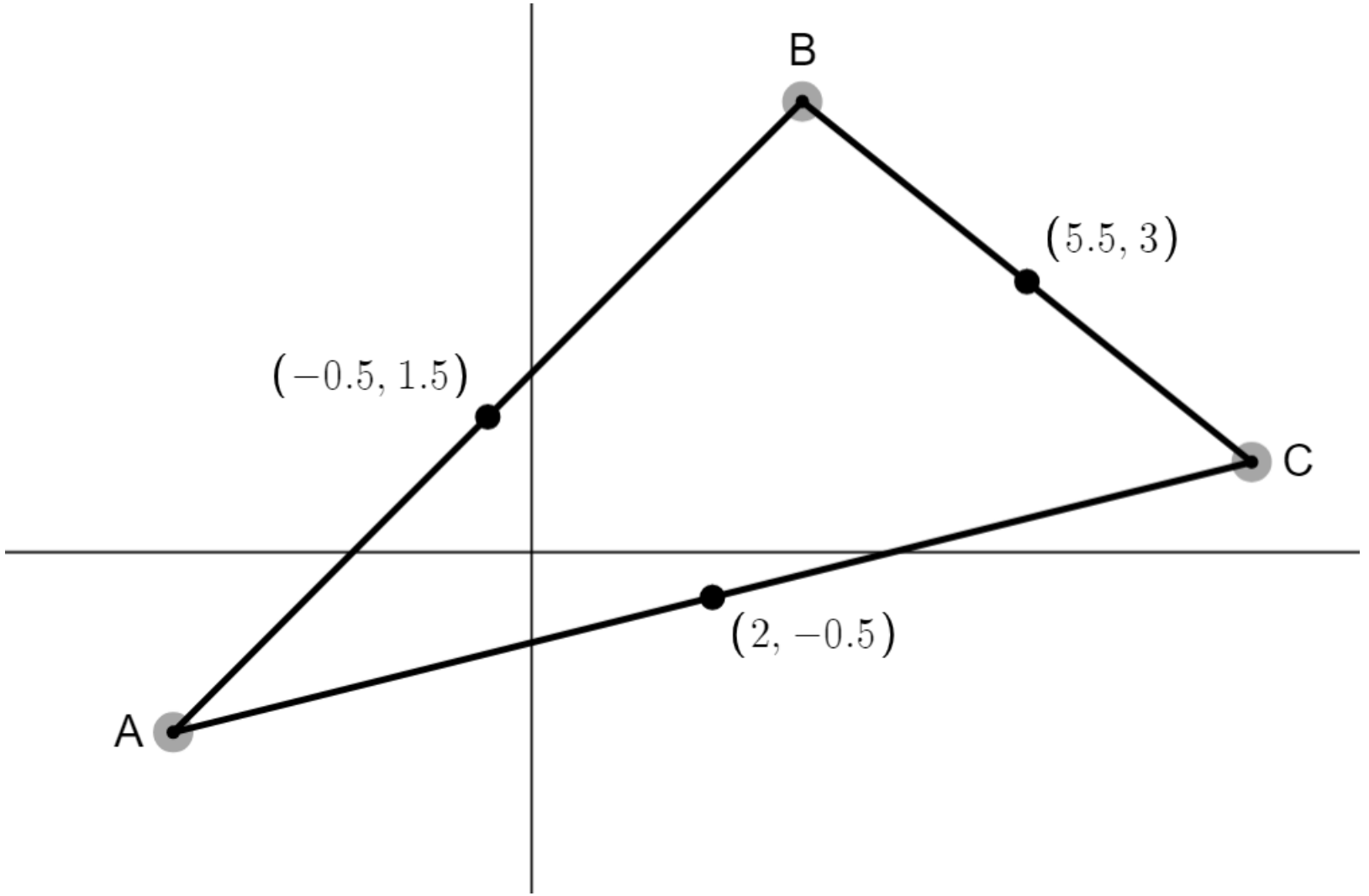
Locates the median for a list of n pieces of ordered data.

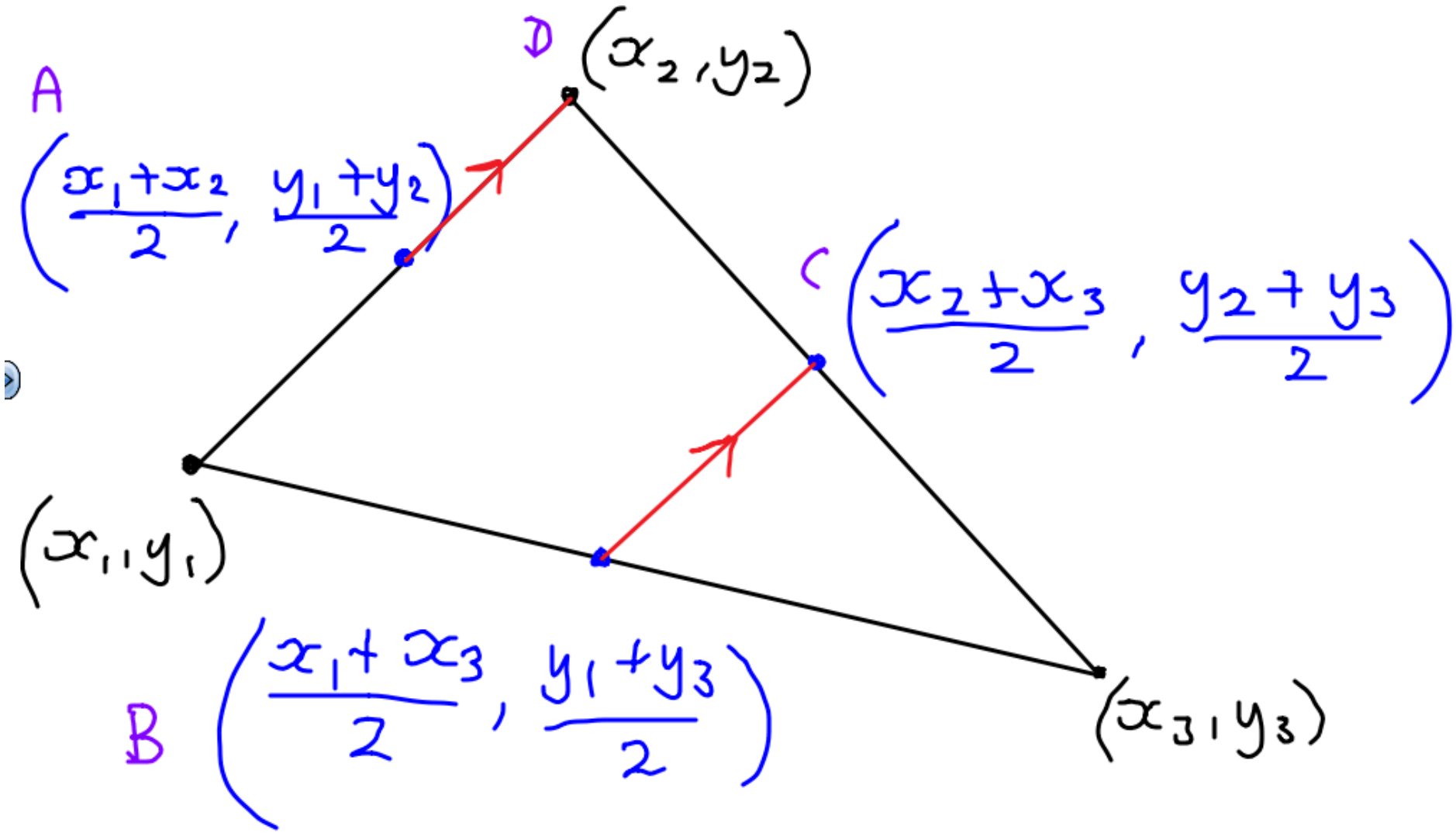
$$\frac{a + l}{2}$$

Finds the "central term" of an arithmetic series.

$$S_n = \frac{n}{2}(a + l)$$

The midpoints of the sides of a triangle are shown. Find the coordinates of its vertices.




D
(x_2, y_2)
A
$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
C
$\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$
(x_1, y_1)
B
$\left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$
(x_3, y_3)

$$\begin{pmatrix} \frac{x_2+x_3}{2} \\ \frac{y_2+y_3}{2} \end{pmatrix} - \begin{pmatrix} \frac{x_1+x_3}{2} \\ \frac{y_1+y_3}{2} \end{pmatrix} = \begin{pmatrix} \frac{x_2-x_1}{2} \\ \frac{y_2-y_1}{2} \end{pmatrix} = \overrightarrow{BC}$$

$$\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} - \begin{pmatrix} \frac{x_1+x_2}{2} \\ \frac{y_1+y_2}{2} \end{pmatrix} = \begin{pmatrix} \frac{x_2-x_1}{2} \\ \frac{y_2-y_1}{2} \end{pmatrix} = \overrightarrow{AD}$$

$\overrightarrow{AD} = \overrightarrow{BC}$ Boom!

Year 12 Pure – Coordinate Geometry.

Fill in the missing information in the table.

Point A	Point B	**Midpoint** of the line segment AB	**Length** of the line segment AB	**Gradient** of the line segment AB	**Equation** of the line through A and B.
$(1,3)$	$(5,11)$				
$(-3,2)$	$(5,-6)$				
$\left(\frac{-7}{3},\frac{-22}{3}\right)$	$\left(\frac{11}{3},\frac{-4}{3}\right)$				
	$(-7,11)$	$\left(-11,\frac{7}{2}\right)$			
	$(-2,-4)$		$2\sqrt{5}$		$x+2y+10=0$
		$(4,1)$	20	$\frac{3}{4}$	
$(4,1)$			$4\sqrt{13}$		$2x+3y-11=0$

To consider:

- Which of these have multiple possible answers?
- If you were not given either point A or point B, what is the minimum information required to complete the row?

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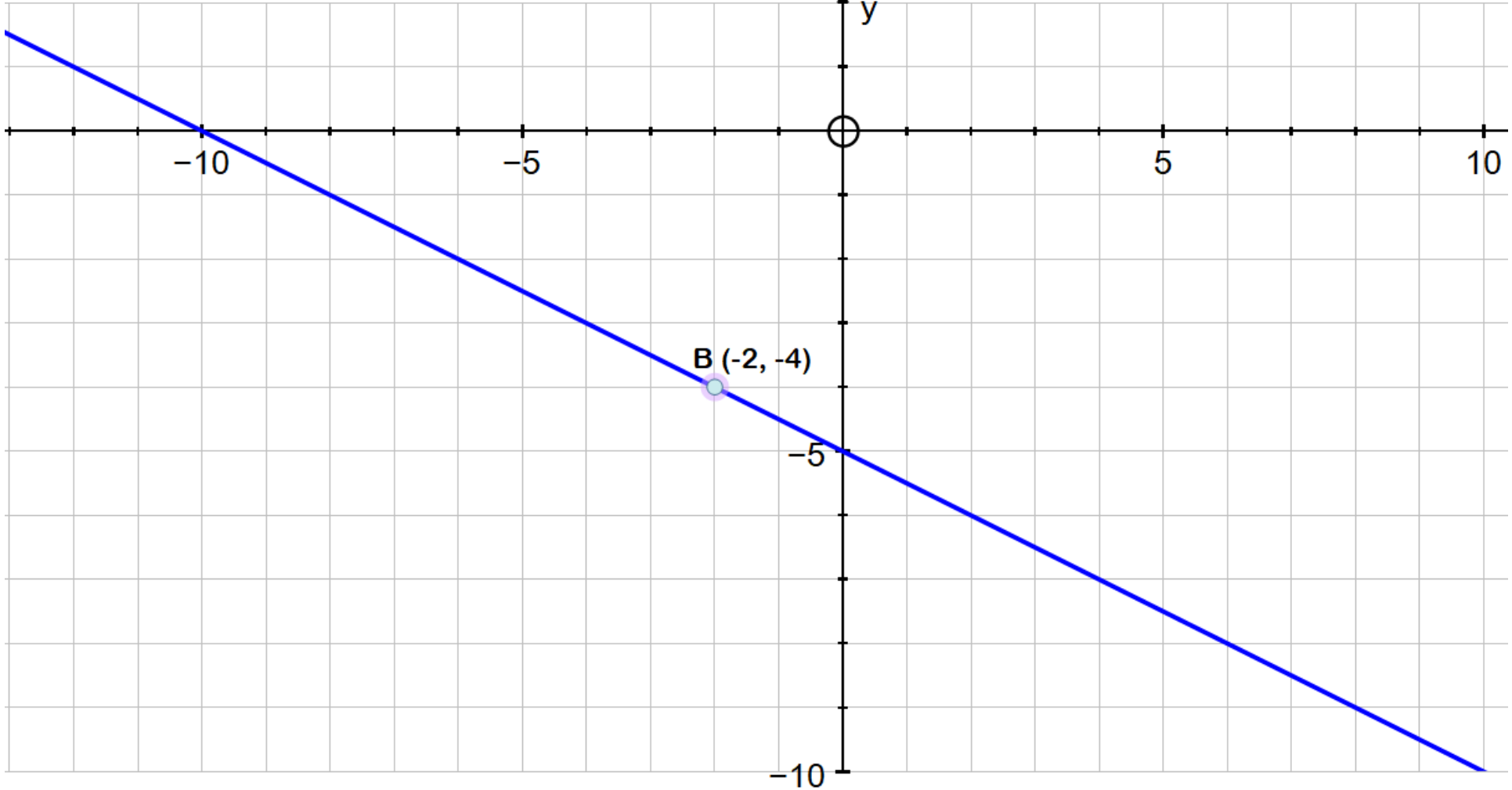

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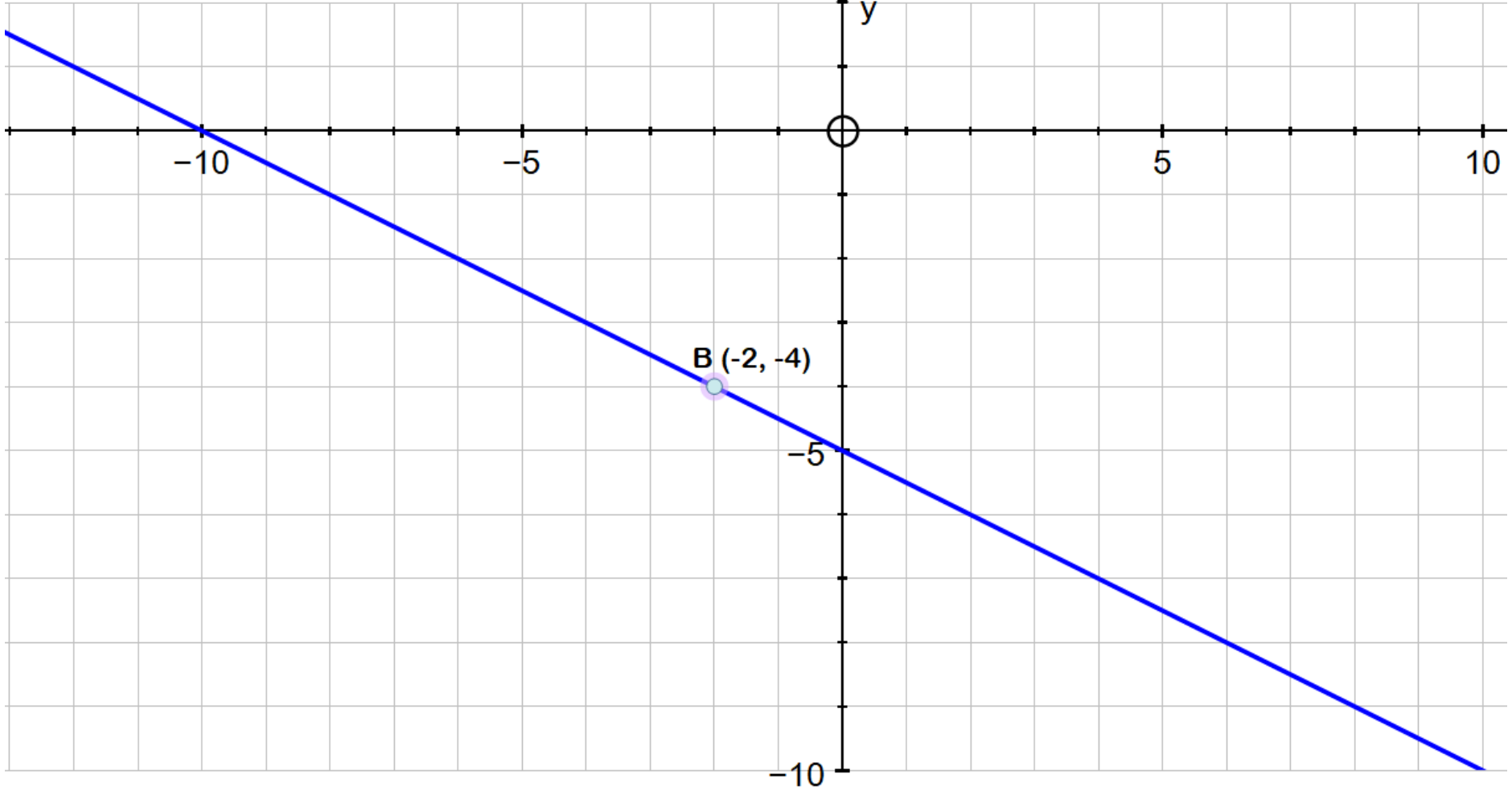


$(x+2)^2+(y+4)^2=\left(2\sqrt{5}\right)^2$

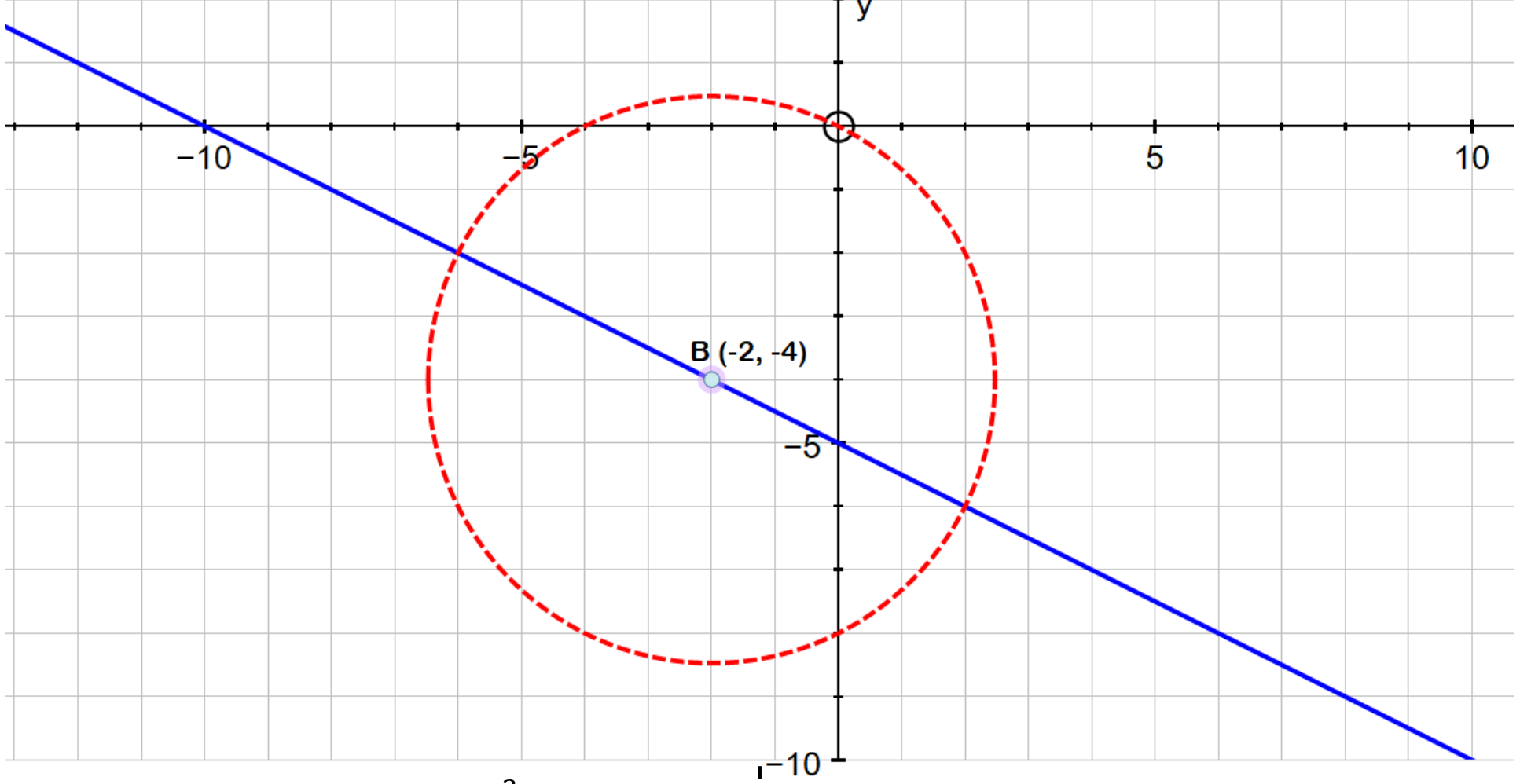
$(x+2)^2+(y+4)^2=20 \quad y=-5-\frac{1}{2}x$

$(x+2)^2+\left(-5-\frac{1}{2}x+4\right)^2=20$

$\frac{5}{4}x^2+5x-15=0$

$x^2+4x-12=0$

$x=-6, \qquad x=2$



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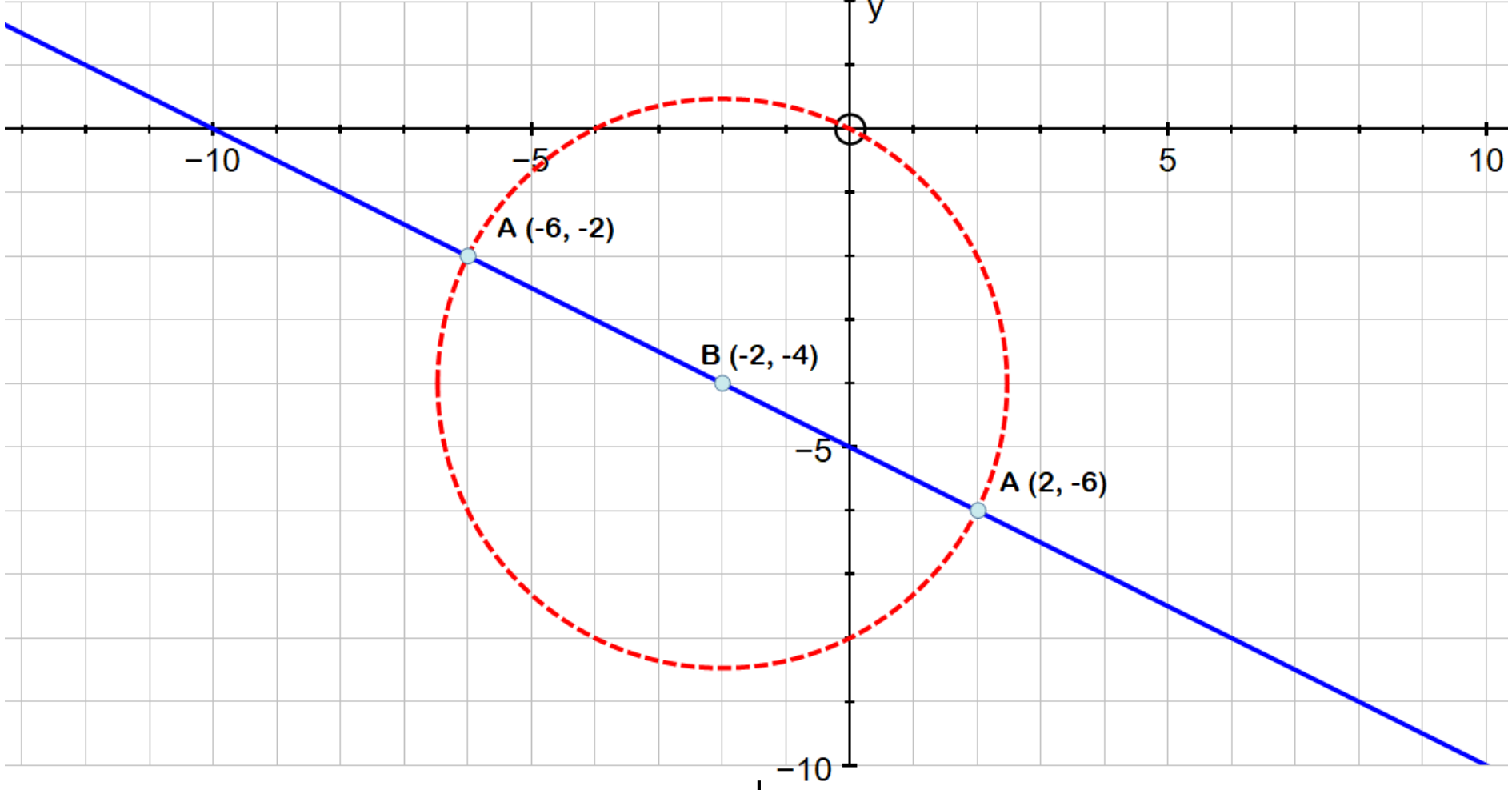

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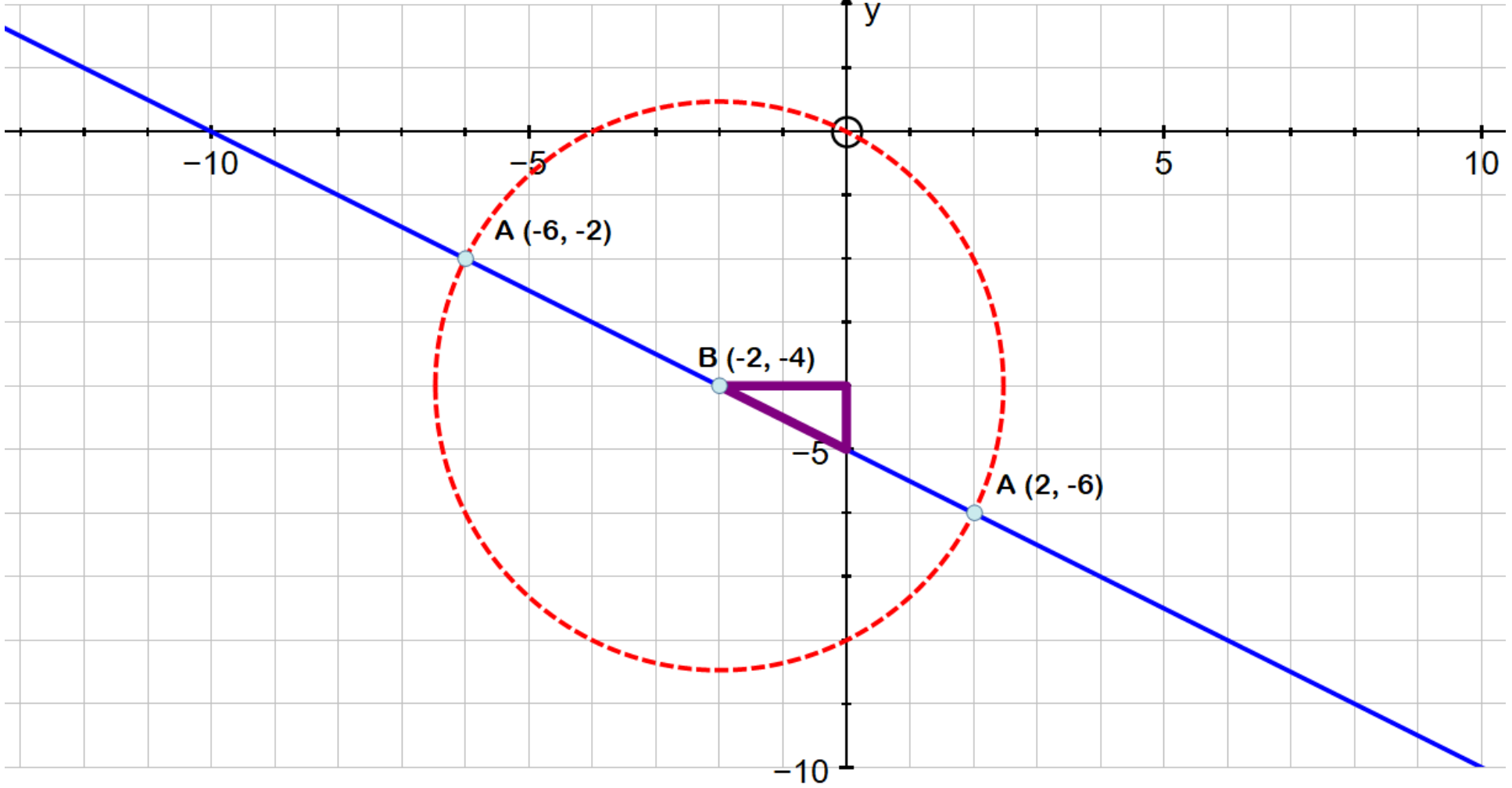
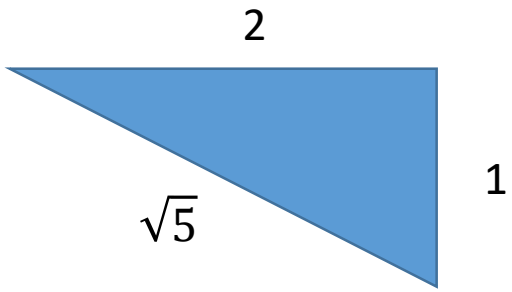
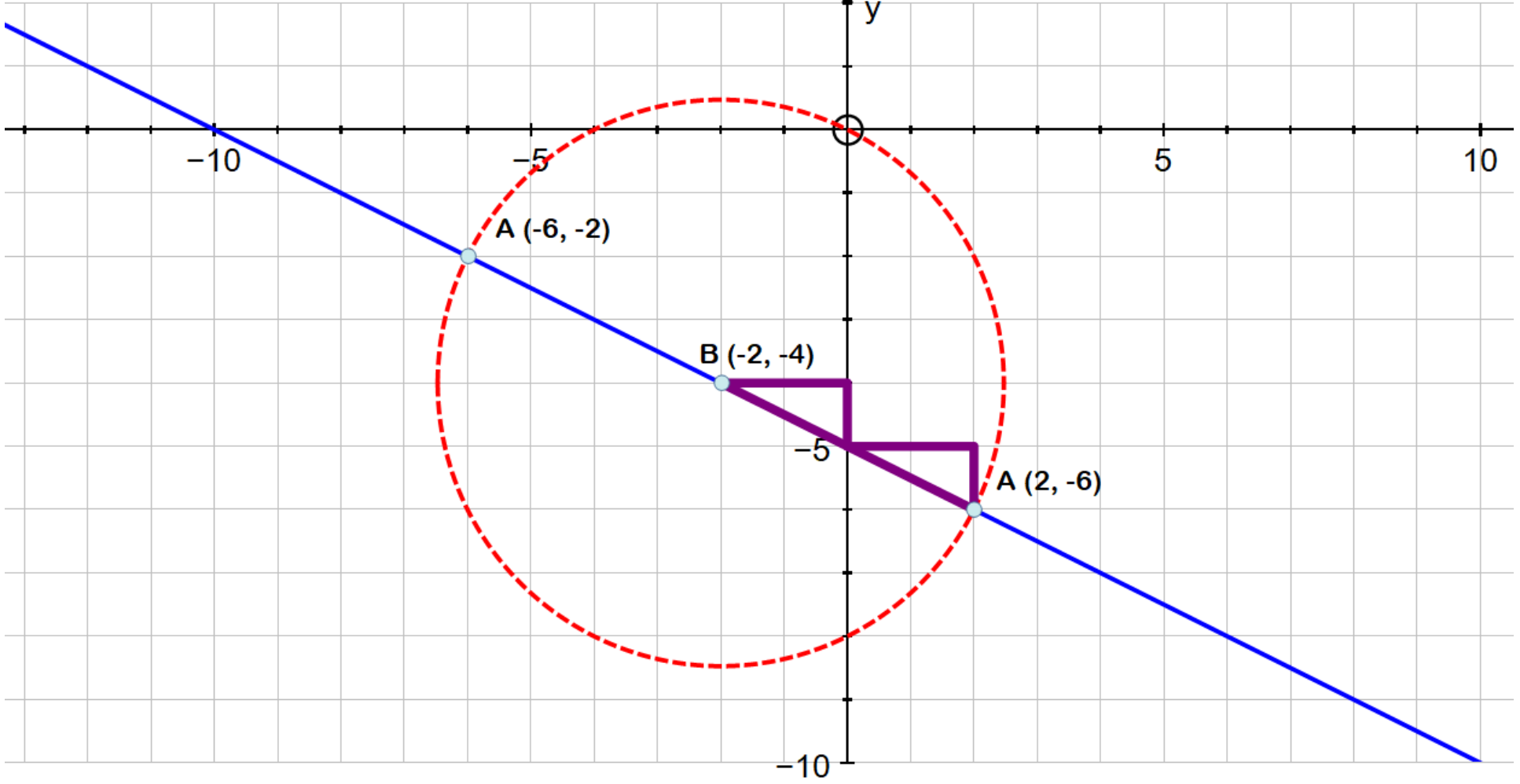

y
−10
−5
5
10
A (-6, -2)
B (-2, -4)
−5
A (2, -6)
−10


2
1
$\sqrt{5}$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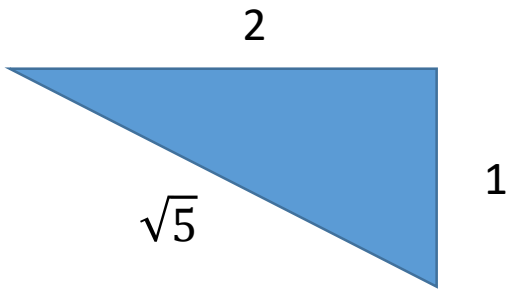
2
1
$\sqrt{5}$

A circle has equation $(x-5)^2+(y+3)^2=10$.
The line l is a tangent to the circle and has gradient -3.
Find two possible equations for l, giving your answer in the form $y=mx+c$.

$y=-3x+c$

$(x-5)^2+(-3x+c+3)^2=10$

Expand

Simplify

Discriminant

A circle has equation $(x-5)^2+(y+3)^2=10$.
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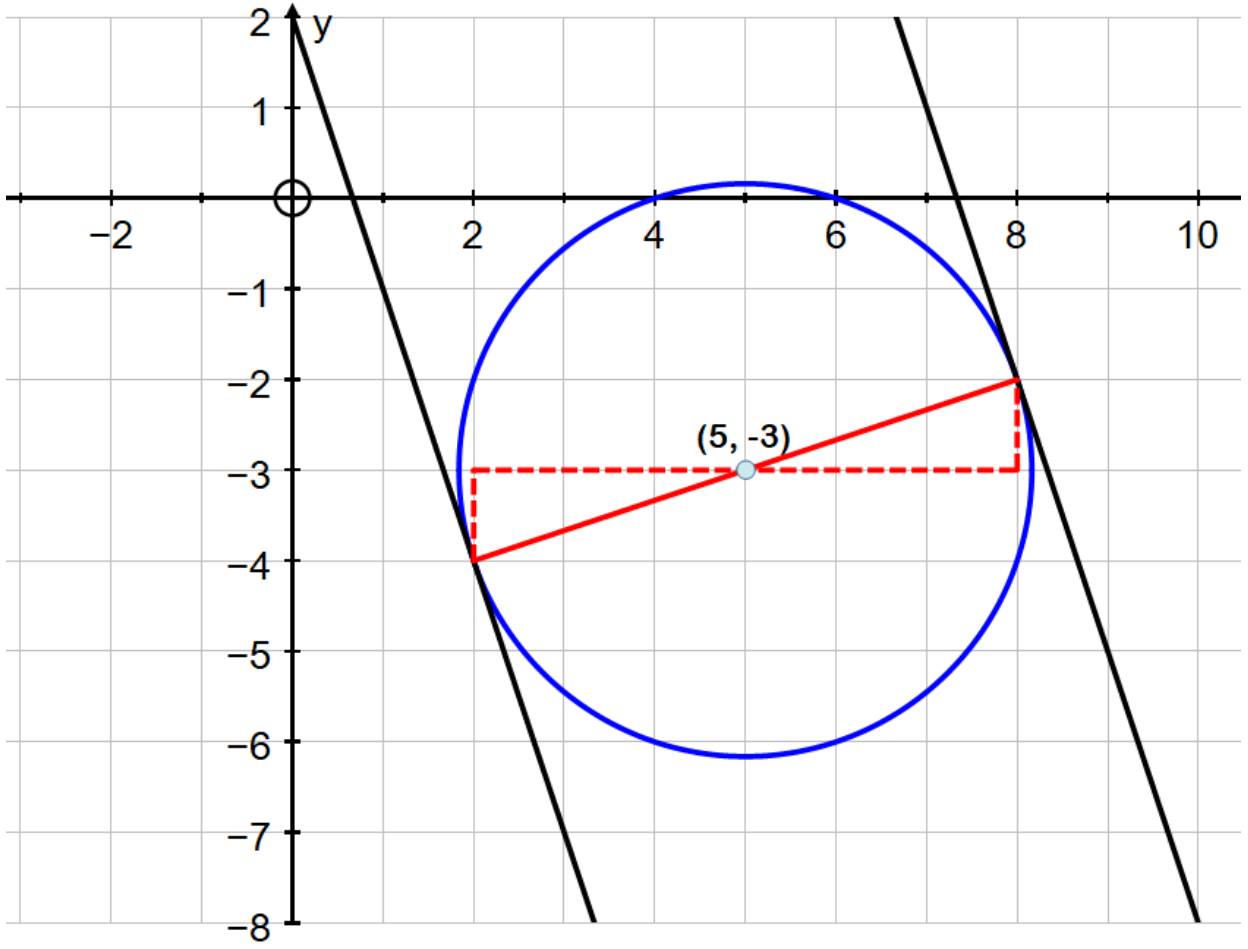
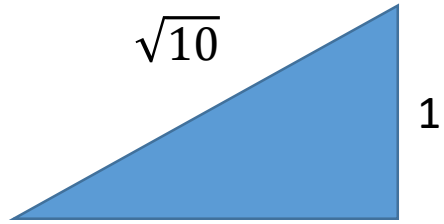




3

Points are
$(8,-2)$ *and* $(2,-4)$

$y=-3x+22$

$y=-3x+2$

Which question is easier?

Question 1

A straight line passes through the points (3,7) and (6,15).
Find the gradient and the y-intercept of the line.

Question 2

An object travelling in a straight line with constant acceleration has a velocity of $7ms^{1}$ after 3 seconds and a velocity of $15ms^{-1}$ after 6 seconds.
Find the acceleration and the initial velocity of the object.

A straight line passes through the points (3,7) and (6,15).
Find the gradient and the y-intercept of the line.

$$m = \frac{15-7}{6-3} = \frac{8}{3}$$

$$y - 7 = \frac{8}{3}(x-3)$$

$$y = \frac{8}{3}x - 1$$

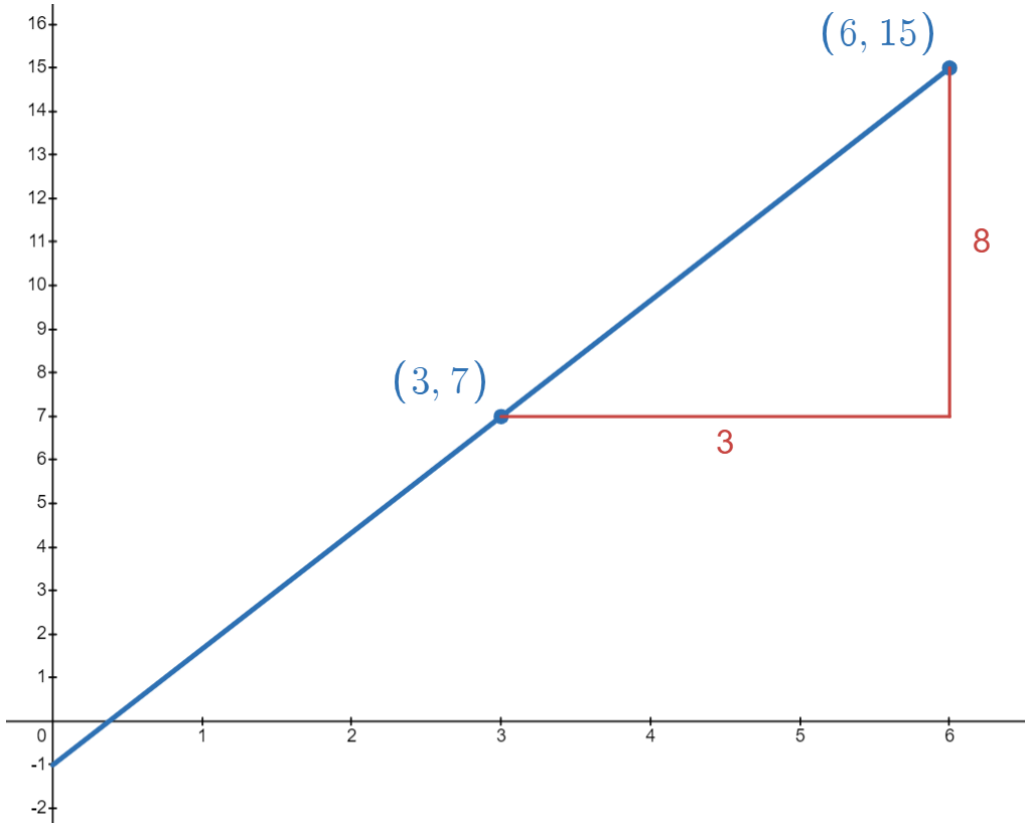
$$m = \frac{8}{3}, c = -1$$

An object travelling in a straight line with constant acceleration has a velocity of $7ms^1$ after 3 seconds and a velocity of $15ms^{-1}$ after 6 seconds.
Find the acceleration and the initial velocity of the object.

$v_1 = u + at_1$ $\quad$ $v_2 = u + at_2$

$7 = u + 3a$ $\quad$ $15 = u + 6a$

$a = \frac{8}{3}ms^{-2}$ $\quad$ $u = -1ms^{-1}$



Which question is easier?

Question 1

The table shows information about the length of flower stems. Use linear interpolation to estimate the median length of a flower stem.

Length of Flower Stem (mm)	Number of Flowers (Frequency)
$28 < l \leq 31$	2
$31 < l \leq 34$	25
$34 < l \leq 37$	30
$37 < l \leq 40$	13

Question 2

A line l_1 passes through the points (34,27) and (37,57).

The line l_2 has equation $y = 35$

Find the x coordinate of the point of intersection of l_1 and l_2.

$$y - 27 = 10(x - 34)$$

$$35 - 27 = 10(x - 34)$$

$$x = 34 + \frac{35 - 27}{10}$$

Length of Flower Stem (mm)	Number of Flowers (Frequency)
$28 < l \leq 31$	2
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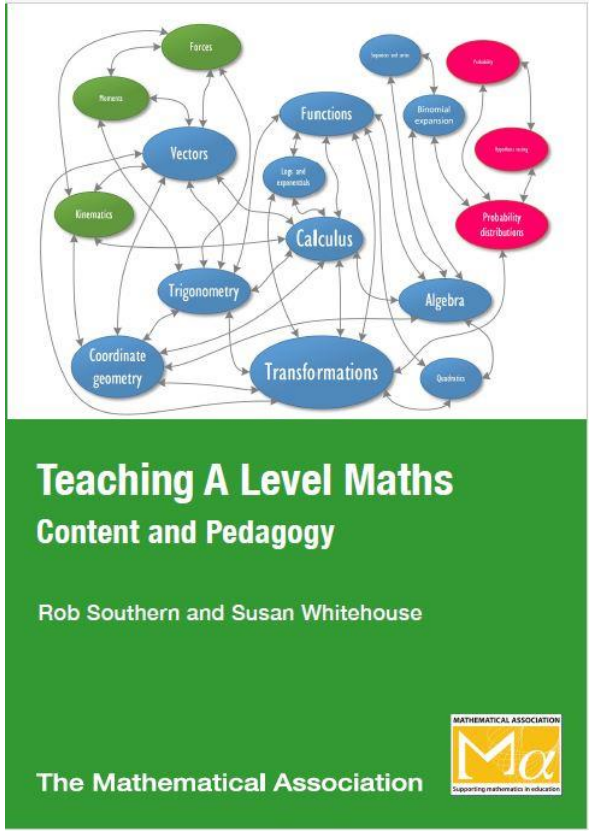
$34 < l \leq 37$	30
$37 < l \leq 40$	13

Teaching A level Maths

Content and Pedagogy

A book by Rob Southern and Susan Whitehouse

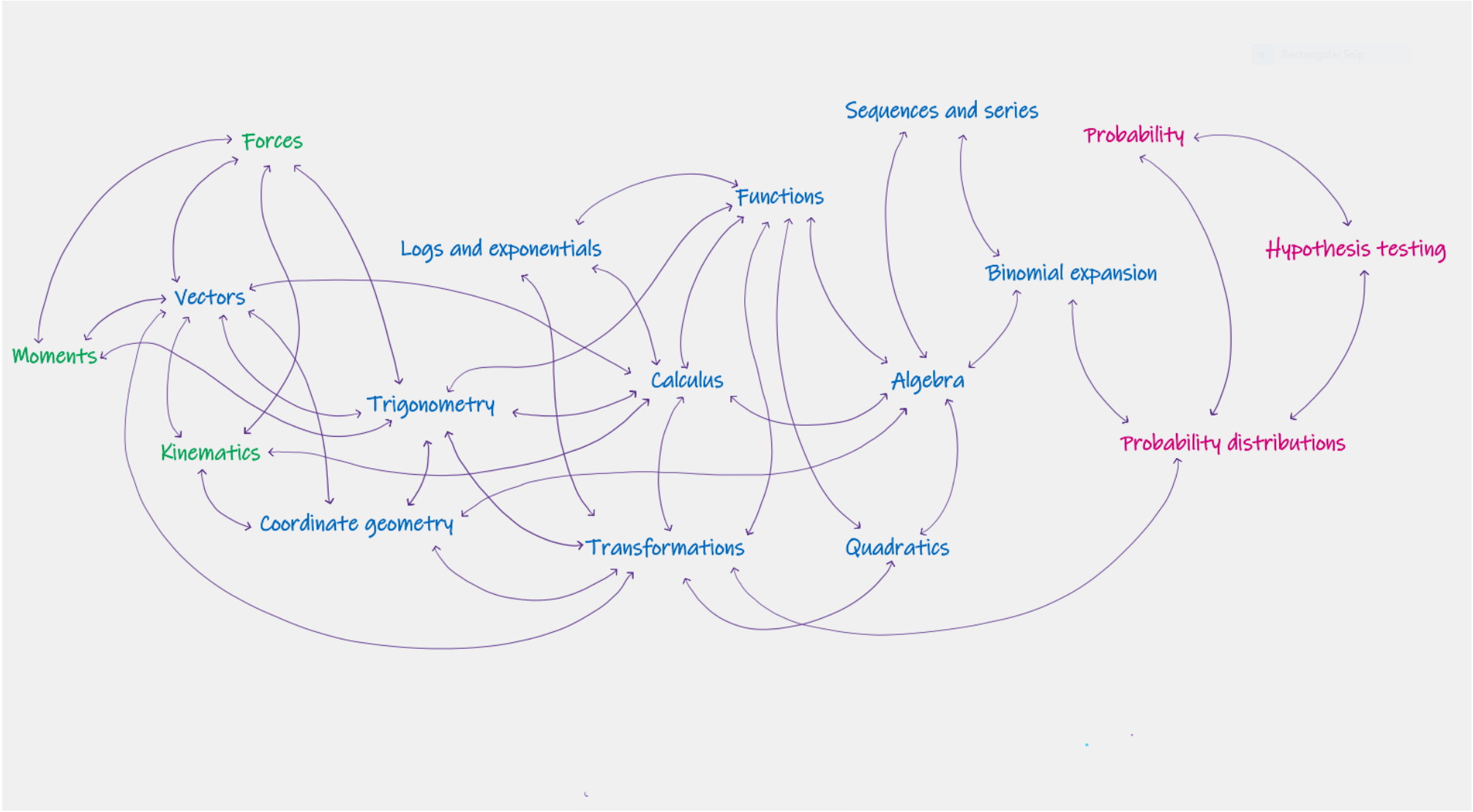


Why did we decide to write this book?

We love teaching A level Maths and discussing ways of teaching A level Maths

We realised that although there are lots of books available about teaching Maths more generally, there were none specifically about teaching A level Maths

Philosophy behind the book



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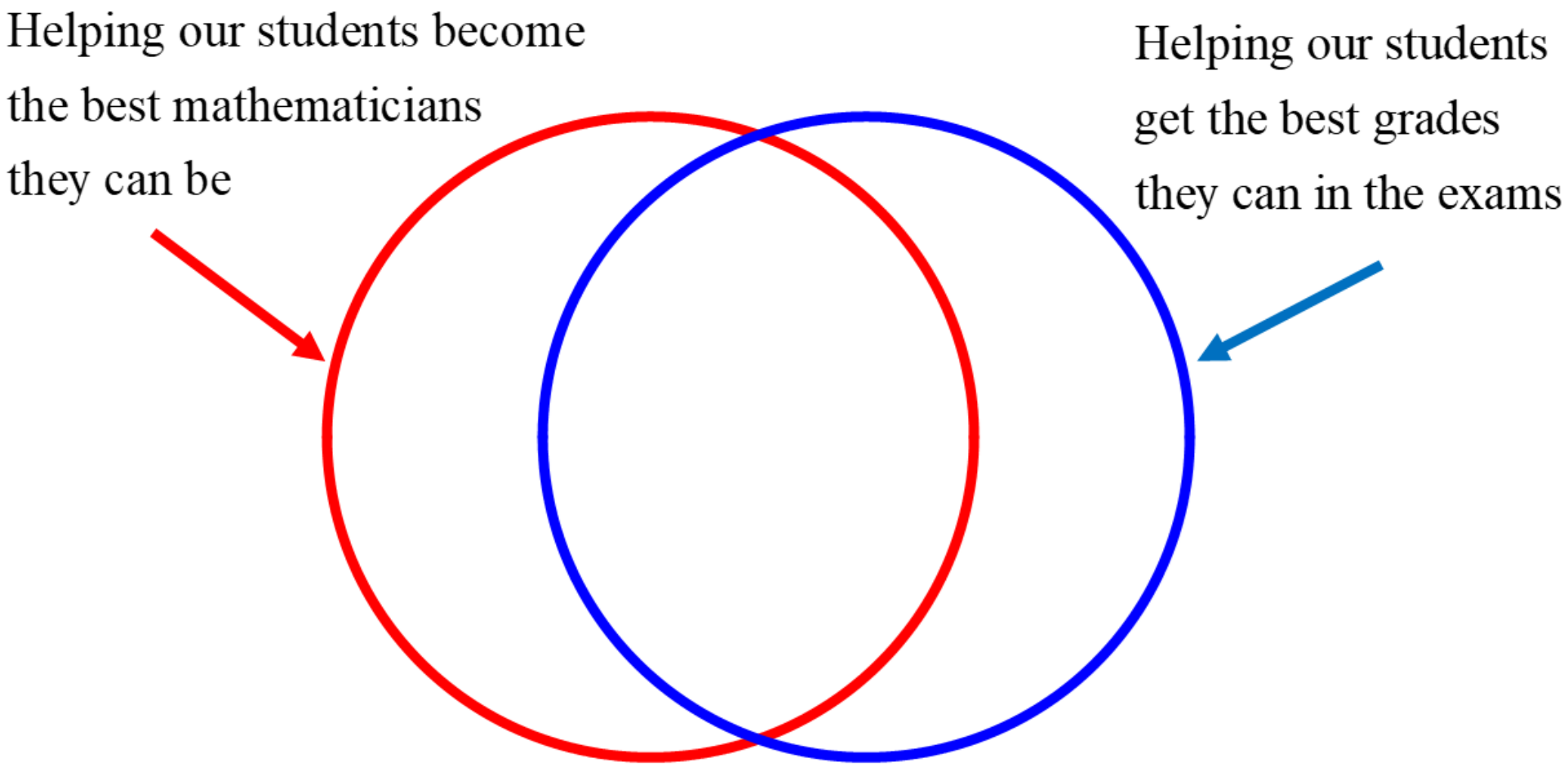

What's in the book?

Some thematic chapters and some topic based chapters

Also, discussion and links to our favourite free resources for each topic

Thematic chapters

- Establishing a mathematical culture in the classroom
- Graphical thinking
- Proof
- Problem-solving and making connections
- Mathematical modelling
- Teaching a topic for the first time
- Curriculum sequencing

Topic based chapters

- Algebra
- Quadratics
- Transformations
- Coordinate Geometry
- Differentiation
- Integration
- Exponentials and logs
- Trigonometry
- Sequences and Series
- Binomial expansion
- Vectors
- Functions
- Probability
- Probability distributions
- Hypothesis testing
- Kinematics
- Forces and Newton’s laws
- Moments

Who's it for?

Especially for teachers new to teaching A level Maths

We hope it will be of interest to experienced A level Maths teachers as well

What it’s not?

It’s not a textbook

Teachers new to teaching A Level Maths should read it alongside a textbook and the specifications

It’s not based on academic research

You’re allowed to disagree with us!

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